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THE LIBRATION OF MERCURY

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ABSTRACT

Integration of the equations of rotation of the planet Mercury is facilated by having a computer perform the pertinent numerical algorithms. The perturbations of the two thirds orbital period rotation of Mercury are calculated accurate to one tenth of an arc second. The numerical solutions to the libratural motion of Mercury are presented.

[redacted]

In a recent report[1], we presented the analysis of the resonance-locked rotation of the planet Mercury under the influence of the planetary potential and the orbital eccentricity. The purpose of the present note is to record the numerical solution of the librational motion. Integration of the equations of rotation of the planet Mercury is facilated by having a computer perform the pertinent numerical algorithms. The perturbations to the two thirds orbital period rotation of Mercury are calculated accurate to one tenth of an arc second.

The equation of the planetary rotation about its polar axis has the form

$$\Phi = f + \phi \tag{1}$$

where f is the true anomaly and ϕ, denoting the angle between the smallest of the moment of inertia with the radius vector, is the solution of the following differential equation[1]

$$\frac{d^2\phi}{df^2} - \frac{2e\sin f}{1+e\cos f}\left(\frac{d\phi}{df}+1\right) + \frac{3\lambda}{1+e\cos f}\cos\phi\sin\phi = 0 \tag{2}$$

in which $\lambda = \dfrac{B-A}{C}$ measures the difference between the two small momentsof inertia and e is the orbital eccentricity.

By applying the Runge-Kutta integration algorithm, equation (2) is solved on an IBM 7090 computer. The computer program performs the necessary algorithms for successively updating expansions of equation (2) truncating only insignificant terms. The over-all precision is consistent with the 10^{-7} truncation level.

In order to investigate the instantaneous physical librations of the planet Mercury, the variable of function Φ has been transformed from the true anomaly f to the mean anomaly M. If E is the planet's eccentric anomaly, a well-known formula of orbital motion

$$\tan\frac{E}{2} = \sqrt{\frac{1-e}{1+e}}\,\tan\frac{f}{2} \tag{3}$$

constitutes the basis of the transformation. Then, by Kepler's equation the mean anomaly can be calculated according to

$$M = E - e\sin E \tag{4}$$

Solutions of Φ are generated for various combinations of parameter λ and initial conditions for a period of 100 years. For the orbital eccentricity of Mercury, the value $e = 0.206$ is adopted. The results of the period and the small amplitude of this librational motion are:

$\lambda = 0.000005$		$\lambda = 0.000050$		$\lambda = 0.000500$	
Period (years)	Amplitude (days)	Period (years)	Amplitude (days)	Period (years)	Amplitude (days)
80	0.008	25	0.008	8	0.008

For large amplitude libration, the period should be corrected by a factor of an elliptic integral. The results indicate that it would be possible to determine the parameter λ for the planet Mercury if the period and the amplitude of its physical libration can be observed.

Since the resonance condition is due to the angle $\Phi - \frac{3}{2}M$ about $\Phi - \frac{3}{2}M = 0$, the rotation of the planet Mercury can remain locked-in motion when the rotational rates are within the range between an upper and a lower limit. A series of computer runs established the following results:

λ	Lower limit for rotational period (days)	Upper limit for rotational period (days)
0.000005	58.538	58.754
0.000050	58.260	59.032
0.000500	57.381	59.911

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REFERENCE

1. Liu, H. S. and J. A. O'Keefe, Theory of rotation for the planet Mercury, Science, 150, 1717, 1965.